

Paper Reference 9FM0/3B
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3B: Further Statistics 1

Friday 14 June 2024 – Afternoon

Time: 1 hour 30 minutes

**DO NOT RETURN THIS QUESTION
PAPER WITH THE ANSWER
BOOKLET AT THE END OF THE
EXAMINATION.**

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Answer Booklet

Data Booklet

Turn over

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Data Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

(continued on the next page)

Turn over

Instructions continued.

Values from statistical tables should be quoted in full. If a calculator is used instead of the tables the value should be given to an equivalent degree of accuracy.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 7 questions in this Question Paper.

The total mark for this paper is 75.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to the table for Question 1 in the Data Booklet.

The discrete random variable X has the probability distribution shown in the table.

(a) Find
 $\text{Var}(X)$
(3 marks)

(b) Find
 $\text{Var}(X^2)$
(3 marks)

(Total for Question 1 is 6 marks)

- 2. The number of errors made by a secretary is modelled by a Poisson distribution with a mean of 2.4 per 100 words.**

A 100–word piece of work completed by the secretary is selected at random.

(a) Find the probability that

(i) there are exactly 3 errors,

(ii) there are fewer than 2 errors.

(2 marks)

(continued on the next page)

Turn over

2. continued.

After a long holiday, a randomly selected piece of work containing 250 words completed by the secretary is examined to see if the rate of errors has changed.

(b) Stating your hypotheses clearly, and using a 5% level of significance, find the critical region for a suitable test.
(4 marks)

(continued on the next page)

Turn over

2. continued.

- (c) Find $P(\text{Type I error})$ for the test
in part (b)
(1 mark)**

(Total for Question 2 is 7 marks)

3. Refer to the table for Question 3 in the Data Booklet.

Tisam took a survey of students' favourite colours.

The results are summarised in the table.

Tisam carries out a suitable test to see if there is any association between favourite colour and year group.

(a) Write down the hypotheses for a suitable test.

(1 mark)

(continued on the next page)

3. continued.

For her table, Tisam only needs to check one cell to show that none of the expected frequencies are less than 5

(b) (i) Identify this cell, giving your reason.

(ii) Calculate the expected frequency for this cell.

(2 marks)

(continued on the next page)

Turn over

3. continued.

**The test statistic for Tisam's test is
38.449**

**(c) Using a 1% level of significance,
complete the test.**

**You should state your critical
value and conclusion clearly.**

(3 marks)

(Total for Question 3 is 6 marks)

4. Every morning Geethaka repeatedly rolls a fair, six-sided die until he rolls a 3 and then he stops.

The random variable X represents the number of times he rolls the die each morning.

- (a) Suggest a suitable model for the random variable X
(1 mark)

- (b) Show that

$$P(X \leq 3) = \frac{91}{216}$$

(2 marks)

(continued on the next page)

Turn over

4. continued.

After 64 mornings Geethaka will calculate the mean number of times he rolled the die.

(C) Estimate the probability that the mean number of rolls is between $5 \cdot 6$ and $7 \cdot 2$ (5 marks)

Nira wants to check Geethaka's die to decide whether or not the probability of rolling a 3 with his die is less than $\frac{1}{6}$

(continued on the next page)

Turn over

4. continued.

Nira rolls the die repeatedly until she rolls a 3

She obtains

$$\mathbf{x = 16}$$

(d) By carrying out a suitable test, determine what Nira's conclusion should be.

You should state your hypotheses clearly and use a 5% level of significance.

(4 marks)

(Total for Question 4 is 12 marks)

Turn over

5. Some of the components produced by a factory are defective.

The management requires that no more than 3% of the components produced are defective.

Niluki monitors the production process and takes a random sample of n components.

(a) Write down the hypotheses Niluki should use in a test to assess whether or not the proportion of defective components is greater than 0.03

(1 mark)

5. continued.

Niluki defines the random variable D_n to represent the number of defective components in a sample of size n . She considers two tests A and B

In test A, Niluki uses $n = 100$ and if $D_{100} \geq 5$ she rejects H_0

**(b) Find the size of test A
(2 marks)**

(continued on the next page)

Turn over

5. continued.

In test B, Niluki uses $n = 80$ and

- if $D_{80} \geq 5$ she rejects H_0**
- if $D_{80} \leq 3$ she does not reject H_0**
- if $D_{80} = 4$ she takes a second random sample of size 80 and if $D_{80} \geq 1$ in this second sample then she rejects H_0 otherwise she does not reject H_0**

(c) Find the size of test B
(3 marks)

(continued on the next page)

Turn over

5. continued.

Given that the actual proportion of defective components is 0.06

(d) (i) find the power of test A

(ii) find the expected number of components sampled using test B

(3 marks)

(continued on the next page)

5. continued.

Given also that, when the actual proportion of defective components is 0.06 , the power of test B is 0.713

**(e) suggest, giving your reasons, which test Niluki should use.
(1 mark)**

(Total for Question 5 is 10 marks)

6. The random variable X has probability generating function $G_X(t)$ where

$$G_X(t) = \frac{1}{\sqrt{4 - 3t}}$$

- (a) Use calculus to find $\text{Var}(X)$

Show your working clearly.

(6 marks)

- (b) Find the exact value of $P(X \leq 2)$
- (4 marks)

6. continued.

The independent random variables X_1 and X_2 each have the same distribution as X

The random variable

$$**Y = X_1 + X_2 + 1**$$

(c) By finding the probability generating function of Y , state the name of the distribution of Y (4 marks)

(continued on the next page)

Turn over

6. continued.

(d) Hence, or otherwise, find

$$\mathbf{P(X_1 + X_2 > 5)}$$

(2 marks)

(Total for Question 6 is 16 marks)

Turn over

7. The probability of winning a prize when playing a single game of Pento is $\frac{1}{5}$

When more than one game is played the games are independent.
Sam plays 20 games.

- (a) Find the probability that Sam wins 4 or more prizes.
(2 marks)

(continued on the next page)

7. continued.

Tessa plays a series of games.

**(b) Find the probability that Tessa
wins her 4th prize on her
20th game.**

(2 marks)

(continued on the next page)

7. continued.

Rama invites Sam and Tessa to play some new games of Pento.

They must pay Rama £1 for each game they play but Rama will pay them £2 for the first time they win a prize, £4 for the second time and £(2w) when they win their Wth prize ($w > 2$)

Sam decides to play n games of Pento with Rama.

(continued on the next page)

Turn over

7. continued.

(c) Show that Sam's expected profit is

$$\text{£} \frac{1}{25} (n^2 - 16n)$$

(6 marks)

Given that Sam chose

$$n = 15$$

(d) find the probability that Sam does not make a loss.

(4 marks)

(continued on the next page)

Turn over

7. continued.

Tessa agrees to play Pento with Rama.

She will play games until she wins r prizes and then she will stop.

(e) Find, in terms of r , Tessa's expected profit.

(4 marks)

(Total for Question 7 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
